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Generalized Magneto-thermoelastic Interaction in a Fiber-Reinforced Anisotropic Hollow Cylinder

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Abstract In this article, an estimation is made to investigate the transient phenomena in the magneto-thermoelastic model in the context of the Lord and Shulman theory in a perfectly conducting medium. A finite element method is proposed to analyze the problem and obtain numerical solutions for the displacement, temperature, and radial and hoop stresses. The boundary conditions for the mechanical and Maxwell's stresses at the internal and outer surfaces are considered. An application of a hollow cylinder is investigated where the inner surface is traction free and subjected to thermal shock, while the outer surface is traction free and thermally isolated. The displacement, incremental temperature, and the stress components are obtained and then presented graphically. Finally, the effects of the presence and absence of reinforcement on the temperature, stress, and displacement are studied.

Keywords Fiber-reinforced · Finite element method (FEM) · Lord and Shulman theory

1 Introduction

Fiber-reinforced composites are used in a variety of structures due to their low weight and high strength. Materials such as resins reinforced by strong aligned fibers exhibit highly anisotropic elastic behavior in the sense that their elastic moduli for extension

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in the fiber direction are frequently of the order of 50 or more times greater than their elastic moduli in transverse extension or in shear. The mechanical behavior of many fiber-reinforced composite materials is adequately modeled by the theory of linear elasticity for transversely isotropic materials, with the preferred direction coinciding with the fiber direction. In such composites the fibers are usually arranged in parallel straight lines. However, other configurations are used. An example is that of circumferential reinforcement, for which the fibers are arranged in concentric circles, giving strength and stiffness in the tangential (or hoop) direction. The theory of strongly anisotropic materials has been extensively discussed in the literature, Belfield et al. [1] studied the stress in elastic plates reinforced by fibers lying in concentric circles. Sengupta and Nath [2] discussed the problem of surface waves in fiber-reinforced anisotropic elastic media. Singh [3] showed that, for wave propagation in fiber-reinforced anisotropic media, this decoupling cannot be achieved by the introduction of the displacement potentials. Hashin and Rosen [4] gave the elastic moduli for fiber-reinforced materials.

During the second half of the twentieth century, non-isothermal problems of the theory of elasticity become increasingly important. This is due to their many applications in widely diverse fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. Second, in the nuclear field, the extremely high temperature and temperature gradients originating inside nuclear reactors influence their design and operations (Nowinski [5]). The theory of couple thermoelasticity was extended by Lord and Shulman (LS) [6] and Green and Lindsay [7] by including the thermal relaxation time in constitutive relations. The theory was extended for an anisotropic body by Dhaliwal and Sherief [8]. Singh [9] studied wave propagation in thermally conducting linear fiber-reinforced composite materials with one relaxation time. Verma [10] discussed the problem of magnetoelastic shear waves in self-reinforced bodies. Chattopadhyay and Choudhury [11] investigated the propagation, reflection, and transmission of magnetoelastic shear waves in a self-reinforced media. Chattopadhyay and Choudhury [12] studied the propagation of magnetoelastic shear waves in an infinite self-reinforced plate. Chattopadhyay and Michel [13] studied a model for spherical SH-wave propagation in self-reinforced linearly elastic media. Abbas and Abd-Alla [14] studied the effect of initial stress on a fiber-reinforced anisotropic thermoelastic thick plate. Abbas et al. [15] studied generalized magneto-thermoelasticity in a fiber-reinforced anisotropic half-space. Tian et al. [16], Abbas [17], Abbas and Abd-Alla [18], and Youssef and Abbas [19] applied the finite element method (FEM) in different generalized thermoelastic problems.

The exact solution of the governing equations of the generalized thermoelasticity theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. To calculate the solution of general problems, a numerical solution technique is used. For this reason the FEM is chosen. The method of weighted residuals offers the formulation of the finite element equations and yields the best approximate solutions to linear and nonlinear boundary and partial differential equations (see Wriggers [20]).

In this article, we have considered a thermal shock problem of generalized magneto-thermoelasticity of a fiber-reinforced anisotropic hollow cylinder. The composite material is then locally transversely isotropic, with the direction of the axis of transverse isotropy not constant, but everywhere directed along the tangents to circles in which the fibers lie. The problem has been solved numerically using a FEM. Numerical results for the temperature distribution, displacement, radial stress, and hoop stress are represented graphically. The results indicate that the effects of a magnetic field, thermal relaxation time, and reinforcement are very pronounced.

2 Basic Equations

The constitutive equations for a fiber-reinforced linearly thermoelastic anisotropic medium whose preferred direction is that of a unit vector **a** [9] are

$$\begin{aligned}\sigma_{ij} &= \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)\\ &\quad \times(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j - \beta_{ij}(T - T_0)\,\delta_{ij},\\ &\quad i, j, k, m = 1, 2, 3, \end{aligned} \tag{1}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad i, j = 1, 2, 3. \tag{2}$$

The Maxwell's stress equation is

$$\tau_{ij} = \mu_e\left[H_i h_j + H_j h_i - H_k h_k \delta_{ij}\right], \quad i, j = 1, 2, 3. \tag{3}$$

The equation of heat conduction under the LS theory is

$$K_{ij} T_{,ij} = \rho c_e\left(\dot{T} + \tau_0 \ddot{T}\right) + T_0 \beta_{ij}\left(\dot{u}_{i,j} + \tau_0 \ddot{u}_{i,j}\right), \quad i, j = 1, 2, 3. \tag{4}$$

The equation of motion is

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i, \quad i, j = 1, 2, 3, \tag{5}$$

where

$$F_i = \left(\vec{J} \times \vec{B}\right)_i, \tag{6}$$

for a slowly moving medium, the variation of the magnetic field and electric field are given by Maxwell's equations:

$$\operatorname{curl} \vec{h} = \vec{J}, \tag{7}$$

$$\operatorname{curl} \vec{E} = -\mu_e \vec{h}, \tag{8}$$

$$\vec{E} = -\mu_e\left(\vec{\dot{u}} \times \vec{H}\right), \tag{9}$$

$$\operatorname{div} \vec{h} = 0, \qquad \operatorname{div} \vec{E} = 0, \tag{10}$$

where ρ is the mass density, u_i is the displacement vector components, e_{ij} is the strain tensor, σ_{ij} is the stress tensor, T is the temperature change of a material particle, T_0 is the reference uniform temperature of the body, β_{ij} is the thermal elastic coupling tensor, c_e is the specific heat at constant strain, K_{ij} is the thermal conductivity, t_0 is the relaxation time, J is the electric current density, μ_e is the magnetic permeability, $\vec{H}$ is the magnetic field vector, λ, μ_T are elastic parameters; α, β, $(\mu_L - \mu_T)$